

Categorization of 31 computational methods to detect spatially variable genes (SVGs) from spatial transcriptomics data

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Spatially Transcriptomics Technologies

Lu Wen, Guoqiang Li, Tao Huang, et al., Single-cell technologies: From research to application, The Innovation, Volume 3, Issue 6, 2022, 100342. https: // doi. org/ 10. 1016/ j. xinn. 2022. 1003/2

Informative features to screen for before linear dimension reduction and Euclidean distance calculation

- HVG detection
	- Used in single-cell transcriptomics data analysis
	- Identifies genes with high expression variability across single cells
	- Helps in clustering cells and identifying subpopulations

• SVG detection

- Used in spatial transcriptomics data analysis
- Identifies genes with high expression variability across spatial locations
- Helps in identifying spatial patterns and regions with distinct molecular signatures

Highly Variable Genes (HVGs) vs. Spatially Variable Genes (SVGs)

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https://www.sc-best-practices.org/_images/svg.jpeg

31 SVG Detection Methods

There is no consensus in SVG definitions

Review

• Adhikari et al., Computational and Structural Biotechnology Journal, 2024 (19 methods)

Benchmark studies

- Charitakis et al., Genome Biology, 2023 (6 methods)
- Chen et al., Genome Biology, 2024 (7 methods)
- Li et al., $bioRxiv$, 2023 (14 methods)

Categorization of SVG definitions is not the focus

Proposal: Three Categories of SVGs

1. Overall SVGs:

- Informative genes for downstream analysis (e.g., spatial domain identification)
- 2. Cell-type-specific SVGs:
	- Revealing spatial variation within a cell type \implies cell subpopulations or states
- 3. Spatial-domain-marker SVGs:
	- Marker genes to annotate and interpret spatial domains already detected

Relationships among the three categories depends on

• Detection methods' null and alternative hypotheses

SVG Categories: Overall, Cell-type-specific, and Spatial-domain-marker SVGs

Categorization of 31 SVG Detection Methods

Hierarchy of 31 SVG Detection Methods (Part 1: Three Categories)

Hierarchy of 31 SVG Detection Methods (Part 2: Overall SVGs)

Hierarchy of 31 SVG Detection Methods (Part 3: Kernel-free Methods)

Hierarchy of 31 SVG Detection Methods (Part 4: Kernel-based Methods)

Hierarchy of 31 SVG Detection Methods (Part 5: Kernel-based Methods)

Hierarchy of 31 SVG Detection Methods (Part 6: Graph-based Methods)

For a given gene with expression levels measured at n spatial spots

Observed variables at spot $i = 1, \ldots, n$

- Gene expression level
	- $v_i \in \mathbb{R}$
	- $Y_i \in \mathbb{R}$: random variable notation
- 2D spatial location
	- $\bullet \;\; \mathsf{s}_i = (s_{i1},s_{i2})^\top \in \mathbb{R}^2$
	- $\bullet\text{ } \textbf{s} = \left[\textbf{s}_1, \ldots, \textbf{s}_n\right]^\top\in \mathbb{R}^{n\times 2}$: spatial location matrix

Inferred variables at spot $i = 1, \ldots, n$

- Spatial-domain indicator vector
	- $\bullet \ \ \mathsf{d}_i = (d_{i1}, \ldots, d_{iL})^{\top} \in \{0,1\}^L$, with $\sum_{l=1}^L d_{il} = 1$
- Cell-type proportion vector
	- $\mathbf{c}_i = (c_{i1}, \dots, c_{iK})^{\top} \in [0, 1]^K$, with $\sum_{k=1}^K c_{ik} = 1$

Among the 31 SVG detection methods, 21 use frequentist inference to detect SVGs:

- Define a test statistic
- Derive the test statistic's null distribution
- Convert the test statistic value to a p-value

Types of null hypotheses:

- Dependence tests: a gene's expression level is independent of spatial location
- Regression-based tests: spatial location has no "effect" on a gene's expression level
	- Fixed-effect tests
	- Random-effect tests (variance component tests)

Null hypothesis:

H_0 : $Y \perp S$

Assume that $(y_1, s_1), \ldots, (y_n, s_n)$ are independently sampled from the distribution of (Y, S)

If H_0 is rejected, the gene is detected as an overall SVG

Nine methods adopt the dependence test formulation:

- Conventional test statistics (with theoretical null distribution): SPARK-X, Hotspot, MERINGUE, BinSpect, scGCO
- Unconventional test statistics (with permutation-based null distribution): Trendsceek, singlecellHaystack, RayleighSelection, SpaGene

SPARK-X (Zhu et al., Genome Biology, 2021)

SPARK-X: a non-parametric test that compares two $n \times n$ spot similarity matrices:

- Matrix 1 based on the gene's expression levels at the *n* spots
- Matrix 2 based on the kernel-transformed spatial locations of the *n* spots

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To detect diverse spatial patterns, SPARK-X transforms the spatial locations $s_i = (s_{i1}, s_{i2})$, $i = 1, \ldots, n$, using two kernel-based functions:

- \bullet Gaussian transformation $s'_{il}=\exp\left(\frac{-s_{il}^2}{2\sigma^2_l}\right)$, $l=1,2$, to detect clustered or focal patterns l
- \bullet Cosine transformation $s'_{il}=\cos\left(\frac{2\pi s_{il}}{\phi_l}\right)$ ϕ $\big), l = 1, 2$, to detect periodic patterns

where σ_1 , σ_2 , ϕ_1 , and ϕ_2 are tuning parameters

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Test statistic: Pearson correlation of the two matrices

Theoretical null: mixture chi-square distribution

singlecellHaystack (Vandenbon and Diez, Nature Communications, 2020)

singlecellHaystack: a unconventional test involves two pre-processing steps:

- Binarize the gene's expression levels at spots into two states: detected and undetected
- Divide the 2D Euclidean space into grid points as coarse spatial coordinates

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singlecellHaystack uses a 2D independent Gaussian kernel, assuming independence of the two dimensions of the Euclidean space, to define **three distributions** of grid points:

- A reference distribution based on all grid points
- A conditional distribution based on grid points in the detected state
- Another conditional distribution based on grid points in the undetected state

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Test statistic: sum of Kullback-Leibler divergences of the two conditional distributions from the reference distribution

Permutation null

Two types: fixed-effect tests and random-effect tests

Linear mixed-effect model (LMM) for a given gene:

$$
Y_i = \beta_0 + \mathbf{x}_i^{\top} \boldsymbol{\beta} + \mathbf{z}_i^{\top} \boldsymbol{\gamma} + \epsilon_i
$$

- \bullet Y_i : a gene's expression level at spot *i* (response variable)
- β_0 : (fixed) intercept
- $x_i \in \mathbb{R}^p$: fixed-effect covariates of spot *i*
- $\beta \in \mathbb{R}^p$: fixed effects

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- β_0 : (fixed) intercept
- $x_i \in \mathbb{R}^p$: fixed-effect covariates of spot *i*
- $\beta \in \mathbb{R}^p$: fixed effects
- $z_i \in \mathbb{R}^{q}$: random-effect covariates of spot *i*
- $\bullet\;\gamma\in\mathbb{R}^{q}\colon$ random effects with zero means $\mathbb{E}[\gamma]=0$ and covariance matrix

 $\text{Cov}(\gamma) \in \mathbb{R}^{q \times q}$

 \bullet ϵ_i : independent random error at spot *i* with $\mathbb{E}[\epsilon_i] = 0$

$$
\bullet\ \gamma\perp\epsilon=(\epsilon_1,\ldots,\epsilon_n)^\top
$$

$$
Y_i = \beta_0 + \mathbf{x}_i^{\top} \boldsymbol{\beta} + \mathbf{z}_i^{\top} \boldsymbol{\gamma} + \epsilon_i
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Fixed-effect tests examine whether x_i contribute to $\mathbb{E}[Y_i]$

If \mathbf{x}_i makes no contribution, then $\mathbb{E}[Y_i|\mathbf{x}_i] = \mathbb{E}[Y_i]$

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Null hypothesis

$$
\mathit{H}_0: \beta = 0
$$

implies $\mathbb{E}[Y_i|\mathbf{x}_i] = \mathbb{E}[Y_i]$, $i = 1, \ldots, n$

$$
Y_i = \beta_0 + \mathbf{x}_i^{\top} \boldsymbol{\beta} + \mathbf{z}_i^{\top} \boldsymbol{\gamma} + \epsilon_i
$$

Random-effect tests examine whether z_i contribute to $\text{Var}(Y_i)$:

$$
\text{Var}(Y_i) = \text{Var}(\mathbb{E}[Y_i|\mathbf{z}_i]) + \mathbb{E}[\text{Var}(Y_i|\mathbf{z}_i)] = \mathbf{z}_i^{\top} \text{Cov}(\boldsymbol{\gamma}) \mathbf{z}_i + \text{Var}(\epsilon_i)
$$

If z_i makes no contribution, then $Var(E[Y_i|z_i]) = 0$

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Null hypothesis

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H_0: \mathrm{Cov}(\gamma)=0
$$

implies $\text{Var}(\mathbb{E}[Y_i|\mathsf{z}_i])=0, i=1,\ldots,n$

Assume
$$
\epsilon_i \stackrel{\text{iid}}{\sim} N(0, \sigma^2)
$$
 and $\gamma \perp \epsilon = (\epsilon_1, ..., \epsilon_n)^\top$

$$
Y_i = \beta_0 + \mathbf{x}_i^\top \boldsymbol{\beta} + \mathbf{z}_i^\top \boldsymbol{\gamma} + \epsilon_i \iff \begin{cases} Y_i \mid \mu_i \stackrel{\text{ind}}{\sim} N(\mu_i, \sigma^2) \\ \mu_i = \beta_0 + \mathbf{x}_i^\top \boldsymbol{\beta} + \mathbf{z}_i^\top \boldsymbol{\gamma} \end{cases}
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Generalized LMM (GLMM): The distribution of Y_i can be non-Gaussian

e.g.,
$$
\begin{cases} Y_i \mid \mu_i \stackrel{\text{ind}}{\sim} \text{Poisson}(\mu_i) \\ \log(\mu_i) = \beta_0 + \mathbf{x}_i^{\top} \beta + \mathbf{z}_i^{\top} \gamma \end{cases}
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Generalized non-parametric mixed-effect model:

The effects of \mathbf{x}_i is modeled as non-parametric:

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\log(\mu_i) = \beta_0 + f(\mathbf{x}_i) + \mathbf{z}_i^{\top} \boldsymbol{\gamma}
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Q: Is spatial location s_i modeled as x_i or z_i ?

Six methods use regression fixed-effect tests, covering all three SVG categories:

- Overall SVGs: SPADE
	- x_i includes s_i
- Cell-type-specific SVGs: C-SIDE, CTSV, and spCV
	- \mathbf{x}_i includes \mathbf{s}_i and \mathbf{c}_i (cell-type proportion vector)
- Spatial-domain-marker SVGs: SpaGCN and DESpace
	- \mathbf{x}_i includes \mathbf{s}_i and \mathbf{d}_i (spatial-domain indicator vector)

SPADE (Bae et al., Nucleic Acids Research, 2021)

SPADE: linear-model fixed-effect test that detects overall SVGs:

$$
\mu_i = \beta_0 + \mathbf{x}_i(\mathbf{s})^\top \boldsymbol{\beta}
$$

• $x_i(s)$: principal components of 512 features from a pre-trained convolutional neural network applied to the n spots' spatial locations s in an H&E image

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Null hypothesis:

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H_0: \boldsymbol{\beta} = \boldsymbol{0}
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If H_0 is rejected, the gene is detected as an overall SVG

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Test: R package limma

(Smyth, G. K., 2005 \Rightarrow Ritchie et al., Nucleic Acids Research, 2015)

spVC (Yu and Li, Genome Biology, 2024)

spVC: fixed-effect test that detects cell-type-specific SVGs

Assume

$$
Y_i \mid \mu_i \stackrel{\text{ind}}{\sim} \text{Poisson}(\mu_i)
$$

Two-step procedure:

1. A reduced model without interactive effects between \mathbf{c}_i and \mathbf{s}_i :

$$
\log(\mu_i) = \beta_0 + \sum_{k=1}^K c_{ik}\beta_k + f_0(\mathbf{s}_i)
$$

It tests two null hypotheses:

- $\bullet \ \ H_0: \bm{\beta} = (\beta_1, \ldots, \beta_{\mathcal{K}})^\top = \bm{0}$ using the likelihood ratio test
- $H_0: f_0(\cdot) = 0$ using the Wald test

If both null hypotheses are rejected, it proceeds to the second step

2. A full model with interactive effects between c_i and s_i :

$$
\log(\mu_i) = \beta_0 + \sum_{k=1}^K c_{ik}\beta_k + f_0(\mathbf{s}_i) + \sum_{k=1}^K c_{ik}f_k(\mathbf{s}_i)
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It tests if any of the interactive effects $f_1(\cdot), \ldots, f_K(\cdot)$ are zero using the likelihood ratio test

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It tests if any of the interactive effects $f_1(\cdot), \ldots, f_K(\cdot)$ are zero using the likelihood ratio test

If

$$
H_0: f_k(\cdot)=0
$$

is rejected, the gene is detected as a SVG specific to cell type k

DESpace (Cai et al., Bioinformatics, 2024)

DESpace: fixed-effect test that detects spatial-domain-marker SVGs Assume

$$
Y_i | \mu_i \stackrel{\text{ind}}{\sim} \text{NegativeBinomial}(\mu_i, \phi)
$$

$$
\log(\mu_i) = \beta_0 + \sum_{l=1}^{L} d_{il} \beta_l
$$

where β_I indicates the effect of spatial domain I

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$$

where β_I indicates the effect of spatial domain I

If

$$
H_0: \beta_I = 0
$$

is rejected, the gene is detected as a marker SVG of spatial domain l

SpatialDE, nnSVG, SOMDE, SVCA, SPARK, and GPcounts

$$
Y_i = \beta_0 + \mathbf{x}_i^{\top} \boldsymbol{\beta} + \mathbf{z}_i^{\top} \boldsymbol{\gamma}(\mathbf{s}) + \epsilon_i
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$$

With n spots, $\textsf{z}_i=(z_{i1},\ldots,z_{in})^\top\in\{0,1\}^n$ is a binary indicator vector for spot i s.t.

$$
z_{ii}=1\,;\quad z_{ij}=0\,\,\text{if}\,\,j\neq i
$$

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Random-effect vector $\boldsymbol{\gamma}(\mathbf{s}) = (\gamma_1(\mathbf{s}_1), \ldots, \gamma_n(\mathbf{s}_n))^\top \in \mathbb{R}^n$ has

 $\gamma_i(\mathbf{s}_i)$ indicating the random effect of \mathbf{s}_i

 $Cov(\gamma(s))$ is assumed to depend on the spatial proximity of s_1, \ldots, s_n via a kernel

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 $Cov(\gamma(s))$ is assumed to depend on the spatial proximity of s_1, \ldots, s_n via a kernel If H_0 : Cov $(\gamma(s))=0$

is rejected, the gene is detected as an overall SVG

SpatialDE (Svensson et al., Nature Methods, 2018)

SpatialDE: a linear random-effect model:

$$
Y_i = \beta_0 + \mathbf{z}_i^{\top} \boldsymbol{\gamma}(\mathbf{s}) + \epsilon_i
$$

- $\bullet\,$ The random errors $\epsilon_1,\ldots,\epsilon_n\stackrel{\textup{iid}}{\sim} \mathcal{N}(0,\delta)$
- The random effects $\gamma(\mathbf{s}) \sim \text{MVN}(\mathbf{0}, \sigma_{\mathbf{s}}^2 \cdot \mathbf{K}(\mathbf{s}))$ The kernel matrix $\mathsf{K}(\mathsf{s}) = [\mathsf{K}(\mathsf{s}_i, \mathsf{s}_j)]_{n \times n}$ is specified by a kernel function $\mathsf{K}(\cdot, \cdot)$

This model is essentially a Gaussian process

If

$$
H_0: \sigma_s^2=0
$$

is rejected, the gene is detected as an overall SVG

26 methods for detecting overall SVGs:

9 kernel-based methods vs. 17 other methods (kernel-free or graph-based)

Kernel-based methods have

- Higher specificity for targeted patterns
- Lower overall power for other patterns

Discussion: Challenges in Detecting Non-Global Expression Patterns

1. Small regions of interests (ROIs)

• Spatial-domain-marker SVGs by first identifying ROIs as spatial domains (e.g., SpaGCN)

2. Spatial-Domain-Specific SVGs

- Genes with spatial patterns in small ROIs but not marker genes
- No existing methods

3. Cell-Type-Specific SVGs

- Easily missed if cell types have small proportions
- Existing methods' model goodness-of-fit

4. Sharp Expression Changes

- Genes with sharp changes at tissue layer boundaries (e.g., Belayer)
- Adding H&E image can help refine tissue boundaries

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Future direction: Incorporate knowledge on "interesting genes" to improve specificity

Discussion: Scalability

- 1. Calculate a summary statistic for each gene.
- 2. Convert the summary statistic to a p-value (frequentist methods only)

Summary Statistic Calulation $(n:$ number of spatial spots)

- Gaussian process: $O(n^3)$ in SpatialDE and SPARK
- Nearest-neighbor Gaussian process approximation: $O(n)$ in nnSVG

p-value Conversion

- Fast if closed-form null distribution is available (conventional statistics)
- Computationally intensive if by permutation (unconventional statistics)

Improving Scalability

- Use approximation algorithms to speed up summary statistic calculation
- Reduce number of permutations in the p-value conversion step

Two Key Differences:

- Spatial Resolution
	- Imaging-based Technologies: Single-cell or subcellular resolution
	- Sequencing-based Technologies: Multicellular level, coarser resolution
- Positional Randomness
	- Structured grids (e.g., Spatial Transcriptomics, 10x Visium)
	- Unstructured spots (e.g., Slide-seq, MERFISH, SeqFISH)

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Current Limitations:

- Most SVG detection methods lack consideration of these technological differences
- Lack of consensus on pre-processing and modeling SRT data

Challenges:

- Double-dipping: Same data analyzed more than once, leading to confirmation bias
- Example: Spatial-domain-marker SVG detection

Strategies:

- Use in silico negative control data to remove spurious discoveries (e.g., ClusterDE)
- Develop fast visualization tools for interpreting top-detected SVGs

Method Benchmarking:

- Benchmarking requires well-annotated datasets with SVG ground truths
- Synthetic datasets and realistic simulators (e.g., SRTsim, scDesign3)
- No method is optimal in every aspect; benchmarking should be specific to data characteristics and align with biological questions

Yan, G., Hua, S. H., & Li, J. J. (2024). Categorization of 31 computational methods to detect spatially variable genes from spatially resolved transcriptomics data. $arXiv$. <https://arxiv.org/abs/2405.18779>